Linear Programming

Assertion & Reason Type Questions

Directions: In the following questions, each question contains Assertion (A) and Reason (R). Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct. The choices are:

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q1.

Assertion (A): Maximum value of $Z = 3x + 2y$, subject to the constraints $x + 2y \leq 2$; $x \geq 0$; $y \geq 0$ will be obtained at point (2, 0).

Reason (R): In a bounded feasible region, it always exist a maximum and minimum value.

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

Q2.

Assertion (A): The linear programming problem, maximise $Z = x + 2y$ subject to the constraints $x - y \leq 10$, $2x + 3y \leq 20$ and $x \geq 0$, $y \geq 0$.

It gives the maximum value of Z as $\frac{40}{3}$.

Reason (R): To obtain the optimal value of Z, we need to compare value of Z at all the corner points of the shaded region.

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

Q3. Assertion (A): Consider the linear programming problem.

Maximise Z = 4x + y

Subject to constraints

$x+y\leq 50, \quad x+y\geq 100$ **and** $x,y\geq 0$

Then, maximum value of Z is 50.

Reason (R): If the shaded region is not bounded then maximum value cannot be determined.

Answer : (d) Assertion (A) is false but Reason (R) is true

Q4.

Assertion (A): The constraints $-x_1+x_2\leq 1$, $-x_1+3x_2\geq 9$ and x_1, $x_2\geq 0$ defines an unbounded feasible space.

Reason (R): The maximum value of $Z=4x+2y$ subject to the constraints $2x+3y\leq 18$, $x+y\geq 10$ and x, $y\geq 0$ is 5.

Answer : (c) Assertion (A) is true but Reason (R) is false

Q5. Assertion (A): For an objective function Z = 15x + 20y, corner points are (0, 0), (10, 0), (0, 15) and (5, 5). Then optimal values are 300 and O respectively.

Reason (R): The maximum or minimum value of an objective function is known as optimal value of LPP. These values are obtained at corner points.

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

Q6. Let the feasible region of the linear programming problem with the objective function Z = ax + by is unbounded and let M and m be the maximum and minimum value of Z, respectively.

Now, consider the following statements.

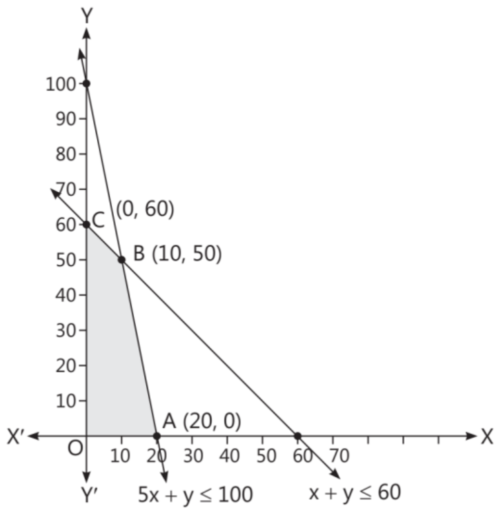
Assertion (A): M is the maximum value of Z, if the open half plane determined by ax + by > M has no point in common with the feasible region. Otherwise, Z has no maximum value.

Reason (R): m is the minimum value of Z, if the open half plane determined by ax + by < m has no point in common with the feasible region. Otherwise, Z has no minimum value.

Answer : (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

Q7. The liner inequalities are:

$$\begin{aligned} 5x + y &\leq 100 && \text{...(1)} \\ x + y &\leq 60 && \text{...(2)} \\ x &\geq 0 && \text{...(3)} \\ y &\geq 0 && \text{...(4)} \end{aligned}$$



Where x and y are numbers of tables and chairs on which a furniture dealer wants to make his profit.

Assertion (A): The region OABCO is the feasible region for the problem.

Reason (R): The common region determined by all the constraints including non-negative constraints x, y ≥ 0 of a linear programming problem.

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

Assertion (A) For an objective function $Z = 15x + 20y$, corner points are (0, 0), (10,0), (0, 15) and (5, 5). Then optimal values are 300 and 0 respectively.

Reason (R) The maximum or minimum value of an objective function is known as optimal value of LPP. These values are obtained at corner points.

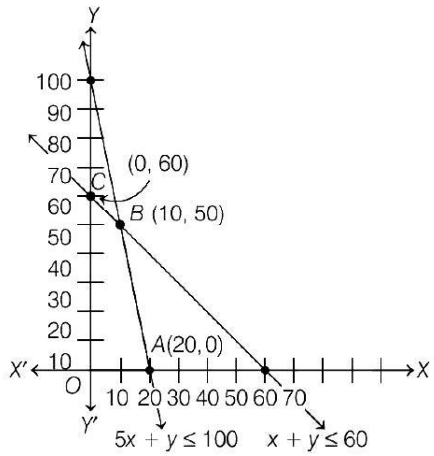
The linear inequalities are

$$5x + y \le 100 \quad \ldots \text{(i)}$$

$$x + y \le 60 \quad \ldots \text{(ii)}$$

$$x \ge 0 \quad \ldots \text{(iii)}$$

$$y \ge 0 \quad \ldots \text{(iv)}$$

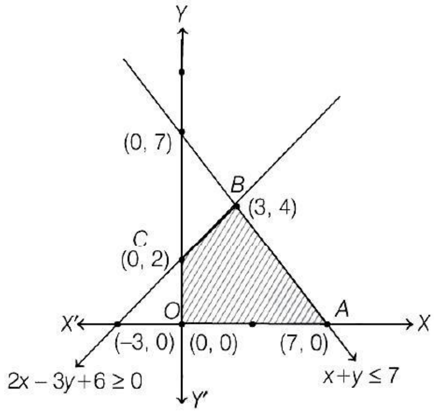


Where x and y are numbers of tables and chairs on which a furniture dealer wants to make his profit.

Assertion (A) The region $OABC$ is the feasible region for the problem.

Reason (R) The common region determined by all the constraints including non-negative constraints $x, y \ge 0$ of a linear programming problem.

Assertion (A) Objective function $Z = 13x - 15y$ is minimised, subject to the constraints $x + y \le 7$, $2x - 3y + 6 \ge 0$, $x \ge 0$, $y \ge 0$.



The minimum value of Z is -21.

Reason (R) Optimal value of an objective function is obtained by comparing value of objective function at all corner points.

Let the feasible region of the linear programming problem with the objective function $Z = ax + by$ is unbounded and let M and m be the maximum and minimum value of Z, respectively.

Now, consider the following statements

Assertion (A) M is the maximum value of Z, if the open half plane determined by $ax + by > M$ has no point in common with the feasible region. Otherwise, Z has no maximum value.

Reason (R) m is the minimum value of Z, if the open half plane determined by $ax + by < m$ has no point in common with the feasible region. Otherwise, Z has no minimum value.

Assertion (A) The maximum value of

$$Z = 11x + 7y$$

subject to the constraints

$$2x + y \le 6$$

$$x \le 2$$

$$x \ge 0,\ y \ge 0$$

occurs at the corner point (0, 6).

Reason (R) If the feasible region of the given LPP is bounded, then the maximum and minimum value of the objective function occurs at corner points.

Assertion (A) Maximum value of $Z = 3x + 2y$, subject to the constraints $x + 2y \leq 2$; $x \geq 0$; $y \geq 0$ will be obtained at point (2, 0).

Reason (R) In a bounded feasible region, it always exist a maximum and minimum value.

Assertion (A) The linear programming problem, maximise $Z = x + 2y$ subject to the constraints $x - y \leq 10$, $2x + 3y \leq 20$ and $x \geq 0$, $y \geq 0$

It gives the maximum value of Z as $\frac{40}{3}$.

Reason (R) To obtain maximum value of Z, we need to compare value of Z at all the corner points of the shaded region.

Assertion (A) Consider the linear programming problem.
Maximise $Z = 4x + y$
Subject to constraints
$x + y \leq 50$, $x + y \geq 100$, and $x, y \geq 0$
Then, maximum value of Z is 50.

Reason (R) If the shaded region is not bounded then maximum value cannot be determined.

Assertion (A) The constraints $-x_1 + x_2 \leq 1$, $-x_1 + 3x_2 \geq 9$ and $x_1, x_2 \geq 0$ defines an unbounded feasible space.

Reason (R) The maximum value of $Z = 4x + 2y$ subject to the constraints $2x + 3y \leq 18$, $x + y \geq 10$ and $x, y \geq 0$ is 5.

Assertion For the given objective function $Z = 15x + 20y$, the corner points table is given below

Corner points	$Z = 15x + 20y$
(0, 0)	0 (minimum)
(10, 0)	150
(0, 15)	300 (maximum)
(5, 5)	175

Optimal value (maximum or minimum) are 300 and 0 from the table.

Reason The maximum or minimum value of an objective function is known as the optimal value of LPP. This is obtained at corner points.

Hence, both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

Assertion In the given figure, the region $OABC$ (shaded) is the feasible region for the problem.

Reason The common region determined by all the constraints including non-negative constraints $x, y \geq 0$ of a linear programing problem is called the feasible region (or solution region) for the problem.

Assertion Shaded region shown as $OABC$ is bounded and coordinates of its corner points are (0, 0), (7, 0), (3, 4) and (0, 2) respectively.

Corner points	Corresponding value of $Z = 13x - 15y$
(0, 0)	0
(7, 0)	91
(3, 4)	-21
(0, 2)	$-30 \leftarrow$ Minimum

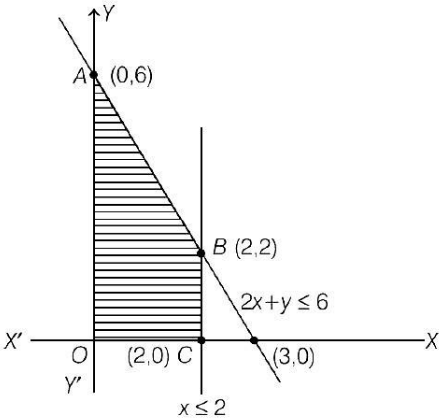
Hence, the minimum value of objective function is at corner point (0, 2) which is -30. Hence, Assertion is not true.

In case, the feasible region is unbounded, we have

Assertion M is the maximum value of Z, if the open half plane determined by $ax + by > M$ has no point in common with the feasible region. Otherwise, Z has no maximum value.

Reason Similarly, m is the minimum value of Z, if the open half plane determined by $ax + by < m$ has no point in common with the feasible region. Otherwise, Z has no minimum value. Hence, Assertion is true and Reason is true but Reason is not the correct explanation of Assertion.

Assertion The corresponding graph of the given LPP is



From the above graph, we see that the shaded region is the feasible region $OABC$ which is bounded.

$\therefore$ The maximum value of the objective function Z occurs at the corner points.

The corner points are O (0, 0), A (0, 6), B (2, 2), C (2, 0).

The values of Z at these corner points are given by

Corner point	Corresponding value of $Z = 11x + 7y$
(0, 0)	0
(0, 6)	42 $\leftarrow$ Maximum
(2, 2)	36
(2, 0)	22

Thus, the maximum value of Z is 42 which occurs at the point (0, 6).

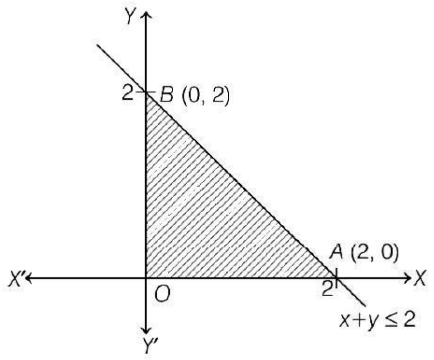
Assertion Given, $x + y \le 2$, $x \ge 0$ and $y \ge 0$

Let $Z = 3x + 2y$

Now, table for $x + y = 2$

x	0	2	1
y	2	0	1

At (0, 0), $0 + 0 \le 2 \Rightarrow 0 \le 2$, which is true.



So, shaded portion is towards the origin.

$\therefore$ The corner points of shaded region are $O(0, 0)$, $A(2, 0)$ and $B(0, 2)$

At point $O(0, 0)$, $Z = 3(0) + 2(0) = 0$

At point $A(2, 0)$, $Z = 3(2) + 2(0) = 6$

At point $B(0, 2)$, $Z = 3(0) + 2(2) = 4$

Hence, maximum value of Z is 6 at point (2, 0).

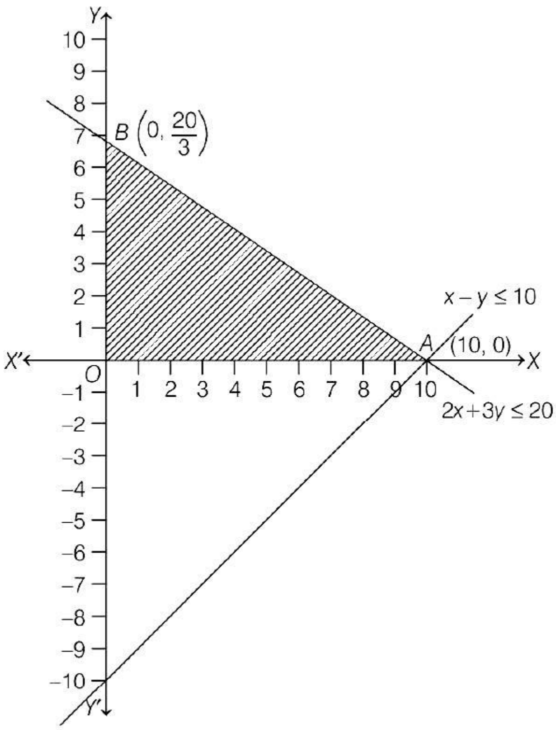
Hence both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

Assertion We have, maximise, $Z = x + 2y$

Subject to the constraints,

$x - y \le 10$, $2x + 3y \le 20$, $x \ge 0$, $y \ge 0$

The graph of constraints are given below



Here, OAB is the required feasible region whose corner points are $O\ (0, 0)$, $A\ (10, 0)$ and $B\left(0, \frac{20}{3}\right)$.

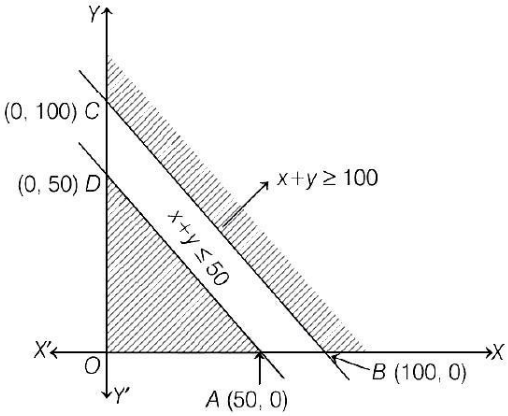
Corner Point	$Z = x + 2y$
At $O\ (0, 0)$	$Z = 0$
At $A\ (10, 0)$	$Z = 10$
At $B\left(0, \frac{20}{3}\right)$	$Z = 0 + 2 \times \frac{20}{3} = \frac{40}{3}$

The maximum value of Z is $\frac{40}{3}$, which is obtained at $B\left(0, \frac{20}{3}\right)$.

Hence, both Assertion and Reason are true and Reason is the correct explanation of Assertion.

Assertion Given, maximise, $Z = 4x + y$

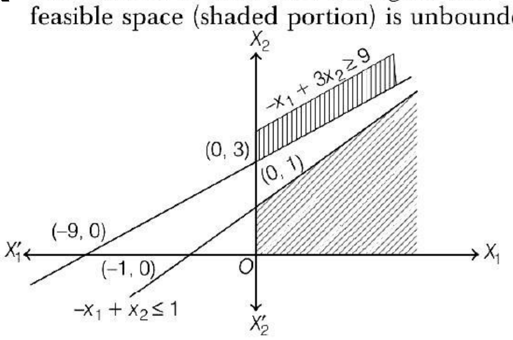
and $x + y \le 50$, $x + y \ge 100$; $x, y \ge 0$



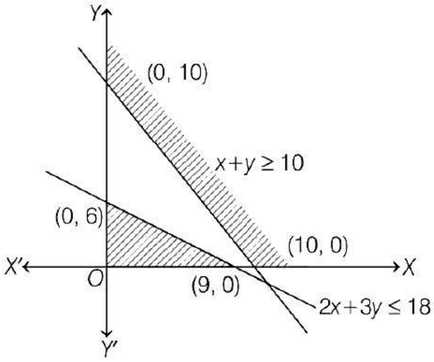
Hence, it is clear from the graph that it is not bounded region. So, maximum value cannot be determined.

Hence Assertion is not true but Reason is true.

Assertion It is clear from the figure that feasible space (shaded portion) is unbounded.



Reason From the figure, it is clear that there is no common area. So, we cannot find maximum value of Z.



Hence, Reason is not true.

Assertion (A): Feasible region is the set of points which satisfy all of the given constraints and objective function too.

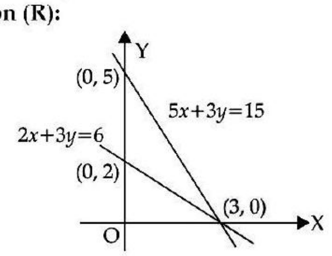
Reason (R): The optimal value of the objective function is attained at the points on X-axis only.

Ans. Option (C) is correct.

> *Explanation:* The optimal value of the objective function is attained at the corner points of feasible region.

Assertion (A): The intermediate solutions of constraints must be checked by substituting them back into objective function.

Reason (R):



Here (0, 2); (0, 0) and (3, 0) all are vertices of feasible region.

Ans. Option (D) is correct.

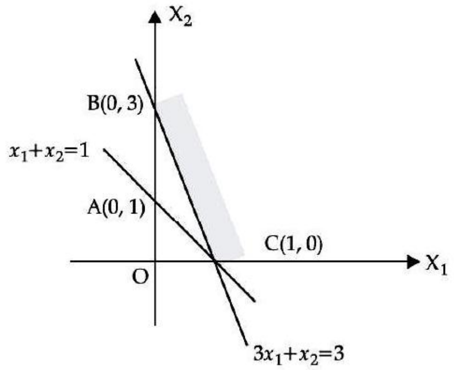
> *Explanation:* The intermediate solutions of constraints must be checked by substituting them back into constraint equations.

Assertion (A) : For the constraints of linear optimizing function $Z = x_1 + x_2$ given by $x_1 + x_2 \leq 1$, $3x_1 + x_2 \geq 1$, there is no feasible region.

Reason (R): $Z = 7x + y$, subject to $5x + y \leq 5$, $x + y \geq 3$, $x \geq 0$, $y \geq 0$. Out of the corner points of feasible region (3, 0), $\left(\frac{1}{2}, \frac{5}{2}\right)$, (7, 0) and (0,5), the maximum value of Z occurs at (7, 0).

Ans. Option (B) is correct.

> *Explanation:* Assertion (A) is correct.
> Clearly from the graph below that there is no feasible region.



Reason (R) is also correct.

Corner Points	$Z = 7x + y$
(3, 0)	21
$\left(\frac{1}{2}, \frac{5}{2}\right)$	6
(7, 0)	49 maximum
(0, 5)	5

Assertion (A): $Z = 20x_1 + 20x_2$, subject to $x_1 \geq 0$, $x_2 \geq 2$, $x_1 + 2x_2 \geq 8$, $3x_1 + 2x_2 \geq 15$, $5x_1 + 2x_2 \geq 20$. Out of the corner points of feasible region (8, 0), $\left(\frac{5}{2}, \frac{15}{2}\right)$, $\left(\frac{7}{2}, \frac{9}{4}\right)$ and (0,10), the minimum value of Z occurs at $\left(\frac{7}{2}, \frac{9}{4}\right)$.

Reason (R) :

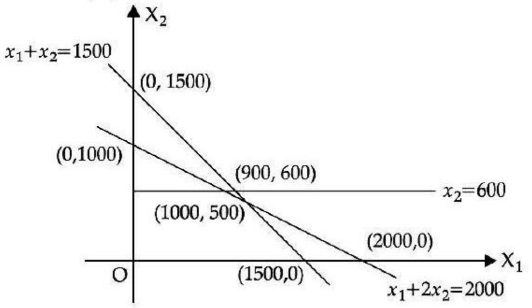
Corner Points	$Z = 20x_1 + 20x_2$
(8, 0)	160
$\left(\frac{5}{2}, \frac{15}{4}\right)$	125
$\left(\frac{7}{2}, \frac{9}{4}\right)$	115 minimum
(0, 10)	200

Ans. Option (A) is correct.

> *Explanation:* Assertion (A) and Reason (R) both are correct and Reason (R) is the correct explanation of Assertion (A).

Assertion (A): For the constraints of a LPP problem given by $x_1 + 2x_2 \leq 2000$, $x_1 + x_2 \leq 1500$, $x_2 \leq 600$ and $x_1, x_2 \geq 0$, the points (1000, 0), (0, 500), (2, 0) lie in the positive bounded region, but point (2000, 0) does not lie in the positive bounded region.

Reason (R):



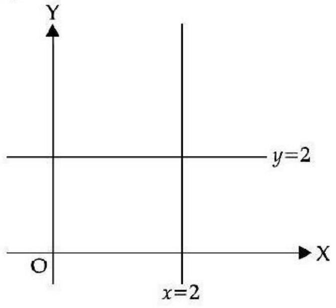
From the graph, it is clear that the point (2000, 0) is outside.

Ans. Option (A) is correct.

> ***Explanation:*** Assertion (A) and Reason (R) both are correct, Reason (R) is the correct explanation of Assertion (A).

Assertion (A) : The graph of $x \le 2$ and $y \ge 2$ will be situated in the first and second quadrants.

Reason (R):



Ans. Option (A) is correct.

> ***Explanation:*** It is clear from the graph given in the Reason (R) that Assertion (A) is true.

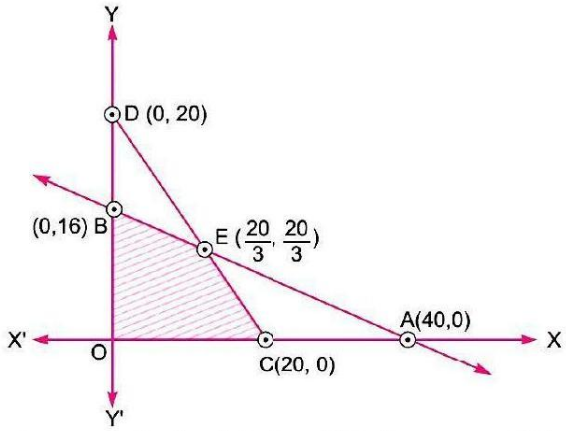
Assertion (A) : The maximum value of $Z = 5x + 3y$, satisfying the conditions $x \ge 0$, $y \ge 0$ and $5x + 2y \le 10$, is 15.

Reason (R) : A feasible region may be bounded or unbounded.

Assertion (A) : The maximum value of $Z = x + 3y$. Such that $2x + y \le 20$, $x + 2y \le 20$, $x, y \ge 0$ is 30.

Reason (R) : The variables that enter into the problem are called decision variables.

Assertion (A) : Shaded region represented by $2x + 5y \ge 80$, $x + y \le 20$, $x \ge 0$, $y \ge 0$ is



Reason (R) : A region or a set of points is said to be convex if the line joining any two of its points lies completely in the region.

Answers